

The Perturbations of Halley's Comet in the Past.—First. Paper.
The period 1301 to 1531.

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We commence this paper by expressing our great indebtedness to Dr. Smart, F.R.A.S.; to Mr. F. R. Cripps, of 22 Hornsey Rise Gardens, N.; and to Mr. Thomas Wright, of 39 Cringle Road, Levenshulme, Manchester; who are really entitled to be considered as joint authors, since they have carried out by far the larger portion of the mechanical quadratures, the results of which are given below. Without their co-operation the completion of the calculations would have been indefinitely delayed; and they are again offering their help in carrying them back to a still more distant date.

The calculation of the perturbations of this comet has already been carried back to 1531 by de Pontécoulant; the identity of the two apparitions before that (those of 1456 and 1378) is universally admitted; before that date Dr. Hind has given (*M.N.*, x. p. 51) a list of conjectured identifications, some fairly certain, like those of 451, 760, 1145, others admittedly vague and uncertain. It occurred to us that new light might be thrown on the question by carrying back the calculation of the perturbations as far as possible, and seeing whether a sufficiently accurate correspondence existed between the conjectured and calculated dates. It is fairly evident that, to bring the labour of computing a large number of revolutions of the comet within reasonable limits, some abbreviations must be introduced, and the following was the plan adopted.

(1) For the purpose of computing the perturbations, the comet in each revolution was assumed to move in an ellipse of constant eccentricity (experience having shown that the perturbations in eccentricity are always small), and with a major axis corresponding to the observed period.

(2) The perihelion and node were supposed to change uniformly from revolution to revolution, the rates of change being deduced from the accepted elements of the comet from 1531 to 1910; subsequently these rates were modified by the use of M. Laugier's discussion (*C.R.*, vol. xxiii.) of the apparitions of 451 and 760, for which fairly definite statements of position are found in the Chinese annals. M. Laugier has been able to represent these observations exactly by elements differing but little from those at the present time. Curiously enough, he does not print his elements, but he gives the longitudes and latitudes calculated from them, and the elements must be very approximately as follows—

(the old system of comet elements is here employed, as more convenient for our purpose):—

T=451	July 3·5	T=760	June 11
$\varpi=284^{\circ}$	} Eq ^x . 451	$\varpi=289^{\circ}$	} Eq ^x . 760
$\Omega=32^{\circ}5$		$\Omega=36^{\circ}5$	
$i=16$		$i=17$	
$q=0^{\circ}60$		$q=0^{\circ}60$	

The following are the collected values of ϖ , Ω , i , reduced to the Equinox of 1910:—

	ϖ	Ω	i
451	304 ^o ·8	53 ^o ·3	16
760	305 ^o ·0	52 ^o ·5	17
1378	306 ^o ·9	54 ^o ·67	17 ^o ·9
1456	305 ^o ·26	50 ^o ·08	17 ^o ·62
1531	306 ^o ·47	50 ^o ·77	17 ^o ·00
1607	305 ^o ·41	52 ^o ·66	17 ^o ·14
1682	305 ^o ·09	54 ^o ·35	17 ^o ·76
1759	305 ^o ·27	55 ^o ·92	17 ^o ·62
1835	305 ^o ·55	56 ^o ·19	17 ^o ·76
1910	303 ^o ·64	57 ^o ·18	17 ^o ·78

Assumed change per re- }
volution of the comet, } - 0^o·06 + 0^o·15

The secular variation of i has been neglected altogether in the period now under discussion, as it appears to be too small to deduce with certainty, considering the roughness of the elements for all the apparitions before 1682.

(3) The co-ordinates of the planets referred to the cometary axes are assumed to repeat themselves at uniform intervals, but the interval for x' , y' is taken slightly different from that for z' , to allow for the adopted motions of the perihelion and the node.

			Period for x', y' .	Period for z' .
			d	d
Venus	...	...	224 ^o ·7005	224 ^o ·7023
Earth	..	...	365 ^o ·2544	365 ^o ·2614
Jupiter	...	...	4332 ^o ·47	4332 ^o ·86
Saturn	...	...	10758 ^o ·6	10760 ^o ·7
Uranus	...	...	30681 ^o ·3	30700 ^o ·6
Neptune	...	...	60167	60238

In consequence of the change in the period and major axis of the comet's orbit, it is more convenient to use the ordinary

astronomical unit of length and the mean solar day as the unit of time. In our previous papers on the perturbations by Jupiter and Saturn 1835-1910 (*M.N.*, lxvii. pp. 386, 511) every quantity tabulated was of zero dimensions in space and time.

(4) In the case of Jupiter and Saturn, the method of mechanical quadratures is used for values of u (the comet's eccentric anomaly) from 0° to 90° , and from 270° to 360° . For the outer half of the orbit the definite integral method described in the above paper (p. 391) is employed; since the eccentricity of the comet's orbit is treated as constant, all the values on p. 409 can be used unchanged except those relating to the position and motion of the perturbing planet; it may be noted that $\frac{x'}{a}, \frac{y'}{a}, \frac{1}{na} \frac{dx'}{dt}, \frac{1}{na} \frac{dy'}{dt}$ vary as $\frac{1}{a}$.

The portion of the perturbations in the outer half of the orbit that we have denoted by V_4 (p. 391) is neglected altogether; this may make an error of a few days in the calculated period, but not sufficient to cause any doubt as to the identity of an observed comet.

The intervals of u for which the calculations are made are $4\frac{1}{2}^\circ$ for Jupiter and Saturn, 9° for Uranus, 18° for Neptune (the calculations for these two planets being made throughout the whole orbit), 1° for the Earth and Venus (the calculations for these two planets being made from the values of u 0° to 30° , 330° to 360°).

(5) We have omitted the discussion of the perturbation of e , Ω , i , having merely found the quantities that come directly into the periodic time.

The following are the assumed co-ordinates of Venus and Earth, referred to the axes of the comet's orbit; they are deduced by a simple transformation from those given by Stratford in the Supplement to the *Nautical Almanac* for 1839, and are in units of the 3rd decimal:—

Julian Day.	Venus.			Earth.		
	x'	y'	z'	x'	y'	z'
2391492	-548	-452	+122	+969	-053	-296
94	569	419	134	966	087	299
96	589	386	146	961	121	302
98	606	352	158	955	154	304
1500	624	317	169	948	187	305
02	638	279	179	941	220	306
04	650	242	189	932	253	307
06	660	204	199	922	286	308
08	668	165	208	910	318	308
10	674	125	216	898	349	308
12	678	085	224	885	380	308
14	679	045	231	871	411	307
2391516	-679	-004	+237	+856	-442	-306

Julian Day.	Venus.			Earth.		
	<i>x'</i>	<i>y'</i>	<i>z'</i>	<i>x'</i>	<i>y'</i>	<i>z'</i>
2391518	- 676	+ 036	+ 242	+ 840	- 472	- 304
20	671	075	247	822	501	302
22	663	115	250	804	530	300
24	654	155	253	785	558	298
26	643	194	255	765	586	295
28	631	233	257	744	613	292
30	615	270	257	723	639	288
32	597	307	256	700	664	284
34	578	343	255	677	689	280
36	557	378	253	653	713	276
38	534	412	250	628	736	271
40	509	444	247	602	758	266
42	483	475	242	576	779	261
44	456	504	237	550	799	255
46	427	532	232	522	819	249
48	397	558	226	493	838	244
50	365	582	218	464	855	236
52	331	604	209	435	871	229
54	297	624	201	405	886	222
56	263	643	192	375	901	214
58	227	660	182	344	914	207
60	191	674	172	312	927	199
62	154	686	161	281	938	191
64	116	696	149	249	948	182
66	078	704	137	217	957	173
68	040	711	124	185	965	164
70	- 002	714	111	152	972	155
72	+ 037	715	098	119	977	146
74	075	714	085	086	981	137
76	113	712	071	052	983	127
78	150	706	057	+ 019	986	117
80	187	698	042	- 014	987	107
82	223	688	028	047	987	097
84	259	676	+ 015	080	985	087
86	294	662	000	114	982	077
88	329	646	- 014	147	978	067
90	362	628	029	179	972	057
92	394	608	044	211	966	046
94	424	586	058	243	958	036
2391596	+ 453	+ 562	- 072	- 275	- 949	- 025

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Julian Day.	Venus.			Earth.		
	x'	y'	z'	x'	y'	z'
2391598	+482	+537	-086	-306	-939	-015
1600	509	510	099	337	928	-004
02	534	481	112	369	916	+007
04	557	451	124	400	903	017
06	578	419	137	430	888	028
08	598	386	149	459	872	038
10	616	352	161	488	855	049
12	632	317	172	515	838	059
14	646	281	183	543	819	070
16	658	244	193	570	799	080
18	668	207	202	596	778	090
20	677	169	211	621	756	100
22	683	130	219	646	734	110
24	687	090	226	670	711	120
26	688	050	233	693	687	130
28	688	+010	239	715	662	140
30	686	-030	244	736	636	149
32	682	069	249	757	609	158
34	675	108	253	777	581	167
36	666	147	256	796	552	176
38	655	186	258	813	523	184
40	643	224	260	829	493	192
42	628	262	260	844	463	200
44	611	299	260	858	432	208
46	593	335	259	871	401	216
48	573	369	257	884	369	223
50	551	403	254	895	337	230
52	527	435	250	905	304	237
54	502	466	246	914	271	243
56	475	495	242	922	237	249
58	446	523	237	929	203	255
60	416	550	231	934	169	260
62	385	574	224	938	135	265
64	353	597	216	941	100	270
66	319	618	208	943	065	275
68	285	638	199	944	-030	279
70	250	655	189	943	+004	283
72	214	670	179	942	039	286
74	177	683	168	940	074	289
2391676	+140	-694	-157	-936	+108	+292

Julian Day.	Venus.			Earth.		
	<i>x'</i>	<i>y'</i>	<i>z'</i>	<i>x'</i>	<i>y'</i>	<i>z'</i>
2391678	+ 103	- 703	- 145	- 931	+ 142	+ 295
80	065	709	133	925	176	297
82	+ 027	713	120	917	210	298
84	- 011	715	107	909	244	299
86	049	715	094	899	278	300
88	087	713	080	888	311	300
90	125	709	066	876	344	300
92	162	702	052	863	376	300
94	199	693	038	849	408	300
96	236	682	023	835	439	299
98	271	669	- 009	819	470	298
1700	305	653	+ 006	802	500	296
02	339	635	020	784	530	294
04	372	616	034	765	559	292
06	403	595	048	745	588	289
08	433	572	062	724	615	286
10	462	547	076	702	642	283
12	489	520	090	680	668	279
14	514	492	103	657	693	275
16	538	462	116	633	717	270
18	- 561	- 431	+ 129	608	741	265
20	...	...	...	582	764	260
22	...	...	...	556	786	255
24	...	...	...	529	806	249
26	...	...	...	502	826	243
28	...	...	...	474	845	237
30	...	...	...	445	863	230
32	...	...	...	416	880	223
34	...	...	...	386	896	216
36	...	...	...	356	911	209
38	...	...	...	326	924	201
40	...	...	...	295	936	193
42	...	...	...	264	948	185
44	...	...	...	232	959	177
46	...	...	...	200	968	169
48	...	...	...	168	976	160
50	...	...	...	136	983	151
52	...	...	...	103	989	142
54	...	...	...	071	994	133
2391756	...	...	...	- 038	+ 997	+ 124

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Julian Day.	Venus.			Earth.		
	x'	y'	z'	x'	y'	z'
2391758	...	...	...	-005	+1000	+114
60	...	...	...	+028	1001	104
62	...	...	...	061	1001	094
64	...	...	...	094	1000	084
66	...	...	...	127	998	074
68	...	...	...	159	994	064
70	...	...	...	191	989	054
72	...	...	...	223	983	044
74	...	...	...	255	977	034
76	...	...	...	287	969	023
78	...	...	...	318	960	013
80	...	...	...	348	949	+003
82	...	...	...	379	938	-008
84	...	...	...	409	926	018
86	...	...	...	438	913	029
88	...	...	...	467	899	039
90	...	...	...	496	883	049
92	...	...	...	524	866	059
94	...	...	...	551	848	069
96	...	...	...	577	829	079
98	...	...	...	603	810	089
1800	...	...	...	629	790	099
02	...	...	...	654	769	109
04	...	...	...	678	747	119
06	...	...	...	701	724	128
08	...	...	...	723	700	137
10	...	...	...	745	676	146
12	...	...	...	766	651	155
14	...	...	...	786	625	164
16	...	...	...	804	598	173
18	...	...	...	821	571	181
20	...	...	...	838	543	189
22	...	...	...	854	515	197
24	...	...	...	870	485	205
26	...	...	...	885	455	213
28	...	...	...	898	425	220
30	...	...	...	910	394	227
32	...	...	...	921	363	234
34	...	...	...	931	332	240
2391836	...	...	...	+940	+300	-246

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Julian Day.	Venus.			Earth.		
	x'	y'	z'	x'	y'	z'
2391838	...	...	...	+948	+268	-252
40	...	...	...	955	235	258
42	...	...	...	961	202	264
44	...	...	...	966	169	269
46	...	...	...	969	136	274
48	...	...	...	972	102	279
50	...	...	...	973	069	283
52	...	...	...	974	035	287
54	...	...	...	973	+002	290
56	...	...	...	971	-032	293
58	...	...	...	968	066	296
60	...	...	...	964	099	299
2391862	...	...	...	+958	-132	-302

The co-ordinates of the comet itself are found from the table (*M.N.*, lxvii. p. 387) by multiplying $\frac{x}{a}, \frac{y}{a}$ by a , using the special value for each revolution, and the number of days from perihelion to each value of u by multiplying nt by $\frac{\text{period in days}}{6.28318}$.

The co-ordinates of Jupiter are found from the table (*M.N.*, lxvii. p. 394) by multiplying $\frac{x'}{a}, \frac{y'}{a}, \frac{z'}{a}$ by 17.7. The following table gives the Julian days corresponding to certain values of g' :—

g'	Julian Day.	g'	Julian Day.
0	2390645	225	2393353
45	1186	270	3894
90	1728	315	4436
135	2269	360	4977
180	2811		

The values for intermediate degrees may be found with sufficient accuracy by reckoning 12^d per degree from the nearest table value. The co-ordinates of Saturn are found from the similar table, referred to (but not printed in detail), *M.N.*, lxvii. p. 512. As the formulæ on which the table is based are given there, the table is not printed here.

As the perturbations by Uranus and Neptune are always small, approximate co-ordinates suffice; those of Uranus are taken from de Pontécoulant (*Mémoire sur le calcul des perturbations de la comète de Halley*, 1829, pp. 37, 38, 48). They are as follows, in units of the first decimal :—

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Julian Day.	x'	y'	z'	Julian Day.	x'	y'	z'
2358000	+187	+26	-55	2369400	-90	-171	+07
300	188	15	57	700	100	165	10
600	189	+04	58	2370000	109	158	14
900	189	-08	59	300	117	151	18
2359200	188	20	60	600	125	143	21
500	187	31	61	900	132	135	25
800	185	42	62	2371200	140	126	28
2360100	182	53	62	500	147	116	31
400	179	64	63	800	153	106	34
700	175	74	63	2372100	158	96	37
2361000	171	84	63	400	162	86	40
300	166	94	62	700	166	75	42
600	160	104	62	2373000	170	64	45
900	154	113	61	300	173	52	47
2362200	147	122	60	600	175	40	49
500	140	131	59	900	175	29	51
800	132	139	58	2374200	176	17	53
2363100	124	147	56	500	176	-05	54
400	116	155	54	800	175	+08	55
700	107	161	53	2375100	174	20	56
2364000	98	167	51	400	172	32	57
300	89	173	49	700	169	44	57
600	79	178	46	2376000	165	56	58
900	69	183	43	300	160	67	58
2365200	59	187	41	600	154	78	57
500	48	190	38	900	148	89	57
800	37	193	35	2377200	141	99	56
2366100	26	195	32	500	133	108	54
400	15	196	28	800	125	118	53
700	+04	196	25	2378100	118	127	52
2367000	-07	196	22	400	109	136	51
300	18	195	18	700	100	144	49
600	29	194	14	2379000	91	151	47
900	40	192	11	300	81	157	45
2368200	50	189	07	600	71	163	42
500	60	186	-04	900	60	169	39
800	70	182	00	2380200	49	173	36
2369100	-80	-177	+03	2380500	-37	+176	+33

Julian Day.	x'	y'	z'	Julian Day.	x'	y	z'
2380800	- 25	+ 179	+ 30	2385000	+ 124	+ 142	- 22
2381100	14	181	27	300	133	135	25
400	- 02	183	24	600	141	127	29
700	+ 09	184	20	900	148	119	32
2382000	21	184	17	2386200	155	110	35
300	32	183	13	500	161	100	38
600	43	181	09	800	167	90	41
900	55	178	05	2387100	172	80	43
2383200	66	175	+ 01	400	176	70	46
500	77	172	- 03	700	180	59	49
800	87	167	07	2388000	183	48	51
2384100	97	162	11	300	185	37	53
400	106	156	14	600	187	26	55
2384700	+ 115	+ 149	- 18	2388900	+ 189	+ 15	- 57

The perturbations produced by Neptune are still smaller than those of Uranus, and, the eccentricity of its orbit being small, it has been considered to move in a circle. The following are its co-ordinates at intervals of 6° of longitude, or 1003 days, in units of the first decimal :—

Julian Day.	x'	y'	z'	Julian Day.	x'	y'	z'
2411217	- 134	- 269	00	2427265	- 240	+ 155	+ 94
2220	160	254	+ 10	8268	222	180	92
3223	184	237	20	9271	202	204	90
4226	206	217	29	30274	179	226	86
5229	226	194	38	1277	154	245	82
6232	244	169	47	2280	128	261	76
7235	259	143	56	3283	100	275	70
8238	271	115	63	4286	071	285	63
9241	280	85	70	2435289	42	293	56
20244	286	55	76	6292	- 12	297	47
1247	288	- 24	82	7295	+ 18	298	38
2250	288	+ 07	86	8298	48	295	29
3253	284	38	90	9301	78	290	20
4256	278	68	92	2440304	107	281	+ 10
5259	268	98	94	1307	134	269	00
2426262	- 256	+ 127	+ 95	2442310	+ 160	+ 254	- 10

The co-ordinates recur with reversed signs after 30090 days.

The dates of perihelion passage in 1378, 1456 were already known with accuracy from the observations; for that of 1301 Hind's value was taken as a working hypothesis, to be tested by the result; the Julian days of the various passages are given here for convenience of reference:—

Perihelion Passage.	Julian Day.	Interval.	Corresponding log α .
1301 Oct. 22·7	2196543·70	^d 28141·07	1·2578
1378 Nov. 8·77	2224684·77	28336·44	1·2598
1456 June 8·21	2253021·21	27471·59	1·2508
1531 Aug. 25·8	2280492·80	27811·07	1·2544
1607 Oct. 26·87	2308303·87	27351·94	1·2496
1682 Sept. 14·81	2335655·81	27936·76	1·2557
1759 Mar. 12·57	2363592·57	28006·37	1·2564
1835 Nov. 15·94	2391598·94	27199·36	1·2480
1910 May 6·3	2418798·30		

The last value is uncertain by several days; the value given was that employed in our Jupiter and Saturn work.

The following are the assumed co-ordinates of Jupiter and Saturn at the times given:—

	u	Jupiter.		Saturn.	
		x'	y'	x'	y'
1301	0°	-4·03	-3·07	-7·07	+5·25
	90	+4·88	+0·28	+5·37	+8·43
	270	-0·48	-4·95	+9·05	+3·63
1378	0	+3·46	+3·97	+3·49	-8·36
	90	-5·02	+0·58	-8·33	-2·49
	270	+1·19	+5·25	-7·44	+4·65
1456	0	-4·58	-2·09	+5·36	+8·44
	90	+4·79	-0·42	+8·51	-3·53
	270	+2·70	-3·92	+6·29	-6·55
1531	0	+0·28	+5·37	-6·60	-5·92

The method pursued has been identical with that used by de Pontécoulant in the memoir quoted above.

$x' - x$, $y' - y$ were found for each value of u ; then $(x' - x)^2 + (y' - y)^2 + z'^2 = \rho^2$, where ρ is the distance between the planet and comet.

Then X , Y , the perturbing forces parallel to the axes, are proportional to

$$\frac{x' - x}{\rho^3} - \frac{x'}{r'^3}, \quad \frac{y' - y}{\rho^3} - \frac{y'}{r'^3}.$$

dn , the alteration in the mean motion, is equal to $X \sin u - Y \cos u \sqrt{1-e^2}$ multiplied by a reducing factor, which is not introduced till after the summation; each value of dn is multiplied by the number of days to elapse before the next perihelion passage.

The following quantities are then formed for each value of u :— $xY - yX$, rX , $r(xX + yY)$. In taking the sums of the columns, only half of the term at the top and bottom of each column is taken; the reason is obvious from the geometry of mechanical quadratures.

The factor for reducing $\int dn$ to seconds of arc is $\frac{6\pi}{365 \cdot 256} a^{\frac{1}{2}} m' du$.

The same factor multiplies the sum of $dn \times$ No. of days to next perihelion passage, and gives a quantity which we call A.

The factor to multiply

$$\int (xY - yX) \sin u - \sqrt{1-e^2} \int rX \text{ is } \frac{m' a du}{e};$$

the result is $\int d\varpi$.

The factor to multiply $\int r(xX + yY)$ is $2 m' du$: this gives a quantity which we call C.

$$\text{Then} \quad \int d\zeta = A - \int d\varpi \sqrt{1-e^2} + C.$$

And period in days is given by the expression

$$\frac{1296000 - \int d\zeta}{n \text{ at beginning of revolution}}.$$

du is the selected interval of u expressed in seconds of arc; thus for interval—

$$\begin{array}{llll} 1^\circ du = 3600, & \text{for } 4\frac{1}{2}^\circ du = 16200, & \text{for } 9^\circ du = 32400, & \text{for } 18^\circ du = 64800 \\ \log = 3.5563 & \log = 4.2095 & \log = 4.5105 & \log = 4.8116 \end{array}$$

The following are the assumed values of $\log m'$ for the different planets :—

$$\text{♂ } 4.3893, \oplus + \text{♂ } 4.4781, \text{♂ } 6.9799, \text{♂ } 6.4558, \text{♂ } 5.6408, \text{♂ } 5.7122; \text{ also}$$

$$\sqrt{1-e^2} = .2540, \text{ i.e. } e = .9672, \quad \log \frac{1}{e} = 0.0145,$$

$$\log \frac{6\pi}{365 \cdot 256} = 8.7127.$$

The following tables exhibit the calculated quantities :—

Revolution 1456 to 1531.

Planet.	Limits of u .	$\int dn$.	A.	$\int d\omega$.	C.	$\int d\xi$.
Venus	0-30	+ "007	+ 200"9	- 0"4	- 1"1	+ 202"1
"	330-360	+ '040	+ 1'3	+ 1'9	0	+ 0'8
Earth	0-30	- '006	- 164'2	- 0'8	+ '7	- 164'7
"	330-360	+ '007	+ 0'6	- 0'4	+ '8	- 0'1
Jupiter	0-90	+ 1'070	+ 29020'	+ 653'9	- 591'4	+ 29445'3
"	90-270	+ '209	...	- 553'4	...	+ 2241'
"	270-360	- '943	- 1105'	- 0'6	- 1290'8	+ 186'0
Saturn	0-90	- '043	- 1096'	+ 49'0	+ 76'9	- 1185'4
"	90-270	+ '050	...	- 128'9	...	+ 790'2
"	270-360	- '018	- 36'6	- 9'4	- 42'6	+ 8'4
Uranus	0-360	'000	+ 0'1	- 4'0	+ 14'8	- 13'7
Neptune	0-360	+ '002	+ 28'9	+ 6'0	+ 25'4	+ 2'0
Sums	...	+ '375	...	+ 12'9	...	+ 31513'

Hence for this revolution we have the equation

$$\frac{1296000'' - 31513''}{n \text{ at } 1456} = 27471'59, \text{ the observed period in days.}$$

Hence n at 1456 = $46''.029$

$$\dots \dots \dots 1531 = 46''.029 + ''0'375 = 46''.404.$$

The last result is in good accord with de Pontécoulant, who gives (*C.R.*, lviii. p. 708) $46''.410$ for the value of n in 1531; as he has computed the perturbations by Jupiter and Saturn only for the revolutions 1531 to 1607, 1607 to 1682, his value is obviously not to be relied on beyond the first decimal.

Revolution 1378 to 1456.

Planet.	Limits of u .	$\int dn$.	A.	$\int d\omega$.	C.	$\int d\xi$.
Venus	0-30	+ "0147	+ 418"7	- 1"4	- 1"1	+ 420"2
"	330-360	- '0019	- 0'9	- 1'5	- '4	- 0'1
Earth	0-30	+ '0148	+ 417'3	- 1'2	- 1'4	+ 419'0
"	330-360	+ '0003	- 0'9	- 2'2	- '3	0'0
Jupiter	0-90	- '1320	- 4065'5	- 611'0	- 187'3	- 3722'
"	90-270	- '182	...	+ 16'4	...	+ 2991'
"	270-360	+ '6725	+ 636'5	- 380'7	+ 841'1	- 108'3
Saturn	0-90	+ '0593	+ 1553'2	- 95'3	- 112'1	+ 1689'5
"	90-270	- '011	...	+ 72'8	...	+ 1586'
"	270-360	- '0240	- 74'5	- 85'6	- 53'4	+ 0'8
Uranus	0-360	- '0013	+ 17'1	+ 1'3	- 1'6	+ 19'0
Neptune	0-360	- '0026	+ 11'2	- 10'5	- 153'0	+ 167'2
* Sums	...	+ '4068	...	- 1101'5	...	+ 3452'3

Hence for this revolution we have the equation—

$$\frac{1296000'' - 3452''\cdot3}{n \text{ at } 1378} = 28336\cdot44, \text{ the observed period in days.}$$

Hence n at 1378 = $45''\cdot614$
. 1456 = $45''\cdot614 + ''\cdot0\cdot407 = 46''\cdot021$.

There are now two independent determinations of n at 1456, which are satisfactorily near each other, considering the many abbreviations that we have introduced; we are justified in adopting their mean, viz. $46''\cdot025$. The corresponding values for 1378, 1531 are $45''\cdot618$, $46''\cdot400$ respectively; the close accord of the latter value with that of de Pontécoulant is satisfactory.

Revolution 1301 to 1378.

Planet.	Limits of u .	$\int dn$.	A.	$\int d\varpi$.	C.	$\int d\xi$.
Venus	0-30	+ ''0023	..	- 2''0	..	+ 63''4
„	330-360	+ ''0020	..	- 1''0	..	0''0
Earth	0-30	+ ''0121	..	- 1''7	..	+ 357''9
„	330-360	+ ''0074	..	+ 2''4	..	+ 0''1
Jupiter	0-90	+ 1''115	+ 30905'	+ 611''2	- 721''2	+ 31471'
„	90-270	+ 1''983	..	- 263''6	..	+ 1303'
„	270-360	- 5980	- 1230'	- 418''8	- 1303''3	+ 180'
Saturn	0-90	+ ''0295	+ 776''4	- 14''6	- 59''8	+ 839''9
„	90-270	- ''040	..	- 106''4	..	- 606''1
„	270-360	+ ''0367	+ 80''9	+ 51''6	+ 76''7	- 8''8
Uranus	0-360	- ''0017	- 7''7	+ 1''0	- 6''9	- 1''1
Neptune	0-360	- ''0001	+ 7''	+ 8''6	+ 28''3	- 23''
Sums	..	+ 763	..	- 133''4	..	+ 33576''3

We now proceed as follows: n at 1301 = $45''\cdot618 - 0''\cdot763 = 44''\cdot855$.

Then theoretical period in days = $\frac{1296000'' - 33576''\cdot3}{44''\cdot855}$
= $28144^d\cdot5$.

This only differs $3^d\cdot5$ from the value obtained by using Hind's identification of the comet of 1301, viz. $28141^d\cdot07$. The dis-

cordance is so small that we are justified in accepting Hind's result with absolute confidence. The long discussion as to the identity of the comet of 1301 is thus happily ended. Dr. Galle, in his *Cometenbahnen*, p. 155, says of the comet of 1378, "Die erste als sicher zu betrachtende Erscheinung des Halley'schen Cometen." We may now claim to have brought the "sicher" returns one revolution further back, and to have thus obtained a firm starting-point for further investigations, which have, in fact, already been commenced.

It will be remembered that Pingré, using the European observations alone, obtained elements quite unlike those of Halley; subsequently Laugier, using a combination of European and Chinese observations, obtained elements resembling those of Halley, except the position of the node, which was 90° greater. Finally, Hind rejected the European observations entirely, and showed that the Chinese ones could be well represented by the Halley elements. Our result indicates that he was justified in this course; it is rather a curious reversal of the present relations of European and Chinese astronomy. Many of Dr. Hind's older identifications rest on Chinese observations, and it is satisfactory to find that the accuracy of the latter in 1378 and 1301 is fully vindicated.

Having once satisfied ourselves of the identity of the comet of 1301, we may rewrite the last equation in the form—

$$\frac{1296000'' - 33576'' \cdot 3}{n \text{ at } 1301} = 28141 \cdot 07, \text{ the observed period in days.}$$

$$\text{Hence } n \text{ at } 1301 = 44 \cdot 861$$

$$\dots\dots\dots 1378 = 44 \cdot 861 + 0 \cdot 763 = 45 \cdot 624.$$

Taking the mean of this and the previously adopted value for 1378, viz. $45 \cdot 618$, we obtain $44 \cdot 858$, $45 \cdot 620$ for the adjusted values in 1301, 1378. It is advisable to make these successive adjustments in order to diminish cumulative error through quantities that we have neglected.

Observations of Comet d 1907, from photographs taken with the 30-in. Reflector of the Thompson Equatorial and the Astrographic 13-in. Refractor at the Royal Observatory, Greenwich.

(Communicated by the Astronomer Royal.)

The following positions of Comet d 1907 were obtained from photographs taken with the 30-in. reflector and the astrographic 13-in. refractor, with exposures of from 20 seconds to 3 minutes.

The plates were measured in the astrographic micrometer. Six reference stars were, as a rule, measured with the comet, their positions being derived from the Catalogues of the Astronomische Gesellschaft.

The positions given are not corrected for Parallax.

Log Parallax Correction = log Parallax Factor - log Δ .

Date and G.M.T. 1907.				Apparent R.A.	Apparent Dec.	Log Parallax Factor R.A.	Dec.
30-in. Reflector.							
d	h	m	s	h m s	° ' "		
June 19	14	3	1	0 16 32.60	+ 1 14 10.8	-9.525	+0.833
July 4	14	15	1	1 14 23.48	5 49 54.7	9.516	0.817
4	14	18	47	1 14 24.23	5 49 57.5	9.512	0.817
16	14	11	38	2 22 8.84	10 28 11.1	9.540	0.805
24	14	14	19	3 22 19.51	13 42 0.7	9.562	0.802
30	14	47	11	4 15 15.97	15 44 19.6	9.564	0.792
31	14	34	8	4 24 26.04	16 0 37.8	9.573	0.799
Aug. 1	14	59	5	4 33 56.40	16 16 0.5	9.565	0.790
6	14	57	39	5 21 17.16	17 8 57.3	9.579	0.800
10	15	12	37	5 58 53.15	17 22 59.5	9.581	0.803
11	15	18	16	6 8 5.99	17 22 42.7	9.581	0.802
12	15	18	36	6 17 10.80	17 20 58.7	9.582	0.805
15	15	32	32	6 43 45.84	17 7 56.9	9.581	0.805
18	15	37	17	7 9 6.29	+16 44 44.8	-9.582	+0.811
Astrographic 13-in. Refractor.							
20	15	46	44	7 25 20.77	+16 24 19.4	-9.581	+0.811

*Royal Observatory, Greenwich :
1907 December 12.*